9/5/19:

M152: Force of Mortality (Hazard Rate)

Notation: $\mu_{x+t} = \mu_x(t) = \mu(x+t)$

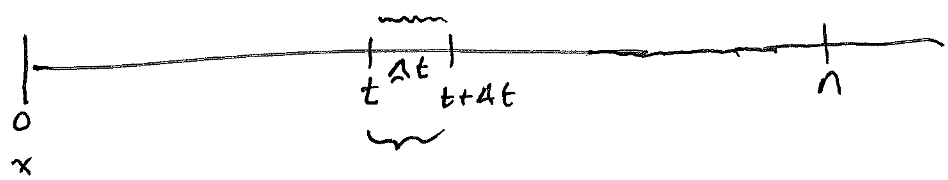
Definition: $$\mu_{x+t} = \frac{f_x(t)}{{}_tp_x}$$

$$\left(\Rightarrow f_x(t) = {}_tp_x \cdot \mu_{x+t}\right)$$

Remarks: 1) $f_x(t) \cdot \Delta t = {}_tp_x \cdot \mu_{x+t} \cdot \Delta t$

$\approx Pr\left((x) \text{ dies between ages } x+t \text{ and } x+t+\Delta t\right)$

${}_nq_x$:



$Pr = \underbrace{{}_tp_x \cdot \mu_{x+t}}_{f_x(t)} \cdot \Delta t$ ↳ becomes dt in the integral below

$$\therefore {}_nq_x = \int_0^n {}_tp_x \cdot \mu_{x+t}\, dt$$

2) $$\mu_{x+t} = \frac{f_x(t)}{{}_tp_x} = \frac{-{}_t\dot{p}_x}{{}_tp_x}$$

Example: Given ${}_tp_x = \left(\frac{100-x-t}{100-x}\right)^2$

Q: (a) $\mu_{30}(20) = ?$

(b) $\mu_{60} = ?$

$\underline{A}$: $\mu_{x+t} = \dfrac{-{}_t\dot{p}_x}{{}_tp_x}$

$$-{}_t\dot{p}_x = 2\left(\frac{100-x-t}{100-x}\right)\left(\frac{+1}{100-x}\right) = \frac{2(100-x-t)}{(100-x)^2}$$

$$\therefore \mu_{x+t} = \frac{2}{100-x-t}$$

$\therefore$ (a) $\mu_{30}(20) \underset{t=20}{\overset{x=30}{=\!=}} \dfrac{2}{50} = .04$

(b) $\mu_{60} \overset{x+t=60}{=\!=} \dfrac{2}{40} = .05$

3) $\mu_{x+t} = \dfrac{-{}_t\dot{p}_x}{{}_tp_x} = -\dfrac{d}{dt}\left[\ln({}_tp_x)\right]$

$$-\int_0^n \mu_{x+t}\,dt = \ln({}_np_x)$$

$$\boxed{\therefore\ {}_np_x = e^{-\int_0^n \mu_{x+t}\,dt} \underset{dy=dt}{\overset{y=x+t}{=\!=}} e^{-\int_x^{x+n}\mu_y\,dy}}$$

<u>Example</u>: Given the following

(i) $\int_0^{10} \mu_{35+t}\,dt = 0.1$

(ii) $\int_{30}^{35} \mu_x\,dx = 0.2$

Determine ${}_{15}q_{30}$.

Solution: From (i) $\quad {}_{10}P_{35} = e^{-\int_0^{10} \mu_{35+t}\,dt} = e^{-0.1}$

and from (ii) $\quad {}_{5}P_{30} = e^{-\int_{30}^{35} \mu_x\,dx} = e^{-0.2}$

$$\therefore \quad {}_{15}P_{30} = {}_{5}P_{30} \cdot {}_{10}P_{35} = e^{-0.3}$$

$$\Rightarrow \quad {}_{15}q_{30} = 1 - e^{-0.3}$$

4) $$\int_0^n {}_tP_x \cdot \mu_{x+t}\,dt = {}_nq_x$$

whereas $\int_0^n \mu_{x+t}\,dt$ leads to ${}_nP_x$ since

$${}_nP_x = e^{-\int_0^n \mu_{x+t}\,dt}$$